

Height and Distance



~ Quantitative Aptitude · SSC Exams ~

← circled = key info!

This whole chapter is just **right-triangle trigonometry** in disguise. Draw the triangle, mark the angle, and reach for **$\tan\theta = \text{height} / \text{base}$** .

① Basic Definitions

Line of sight → the straight line from the observer's eye to the object viewed.

Horizontal line → the level line through the eye, parallel to the ground.

Angle of ELEVATION → angle the line of sight makes **above** the horizontal (looking **up**).

Angle of DEPRESSION → angle the line of sight makes **below** the horizontal (looking **down**).

Elevation (look up)

Depression (look down)

Key link → the angle of **depression** from A (top) to B (ground) **equals** the angle of **elevation** from B up to A – they are **alternate angles** between two parallel horizontals.

Worked example – alternate angles

A man at the top of a **30 m** cliff sees a boat at an angle of **depression 45°**.

Idea → depression 45° = elevation 45° from the boat. In that triangle $\tan 45^\circ = h/d = 1$.

So → $d = h = \mathbf{30\ m}$. The boat is 30 m from the foot of the cliff.

page 1

② The Tool Kit – $\tan\theta$

In a right triangle the angle sits at the observer. The **vertical** is opposite it, the **horizontal** is adjacent.

Master formula

- $\tan\theta = \text{opposite} / \text{adjacent} = \text{height} / \text{base}$
- $\sin\theta = \text{opposite} / \text{hypotenuse}$ • $\cos\theta = \text{adjacent} / \text{hypotenuse}$
- So $\text{height} = \text{base} \times \tan\theta$ and $\text{base} = \text{height} / \tan\theta = \text{height} \times \cot\theta$

Standard angle values – memorise!

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$
30°	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$	$\sqrt{3}$
45°	$1/\sqrt{2}$	$1/\sqrt{2}$	1	1
60°	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$	$1/\sqrt{3}$



Numbers to keep on your fingertips $\rightarrow \sqrt{3} \approx 1.732$, $1/\sqrt{3} \approx 0.577$, $\sqrt{2} \approx 1.414$. Rationalise:
 $1/\sqrt{3} = \sqrt{3}/3$.

Worked example – read the triangle

A pole casts a right triangle of **height 50 m** over a **base of 50 m**. Find the elevation.

Step 1 – $\tan\theta = \text{height} / \text{base} = 50 / 50 = 1$.

Step 2 – $\tan\theta = 1 \Rightarrow \theta = 45^\circ$. $\therefore$ equal height & base always means 45° . 

✓ Complementary pairs $\rightarrow \tan 30^\circ \cdot \tan 60^\circ = (1/\sqrt{3})(\sqrt{3}) = 1$. In general $\tan\theta \cdot \tan(90^\circ - \theta) = 1$.

Quick reads

base 10, $\theta = 30^\circ \rightarrow h = 10/\sqrt{3} \approx 5.77$

base 10, $\theta = 60^\circ \rightarrow h = 10\sqrt{3} \approx 17.32$

base 10, $\theta = 45^\circ \rightarrow h = 10$

h 10, $\theta = 30^\circ \rightarrow \text{base} = 10\sqrt{3} \approx 17.32$

③ Single-Triangle Problems

One angle, one distance

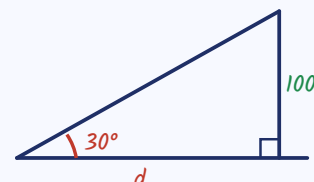
- height $h = d \cdot \tan\theta$ • distance $d = h \cdot \cot\theta = h / \tan\theta$
- If $\theta = 45^\circ$ then $\tan\theta = 1 \Rightarrow \text{height} = \text{distance}$.

Worked example 1

A tower is **100 m** tall. From a point on the ground the elevation of its top is **30°**. How far is the point from the base?

Set up $\rightarrow \tan 30^\circ = h/d \Rightarrow 1/\sqrt{3} = 100/d$.

Solve $\rightarrow d = 100\sqrt{3} \approx \mathbf{173.2 \text{ m}}$. ✓



Worked example 2 – the 45° shortcut

From a point **60 m** from the foot of a tower the elevation of the top is **45°**. Find the height.

Solve $\rightarrow h = d \cdot \tan 45^\circ = 60 \times 1 = \mathbf{60 \text{ m}}$. (height = distance). ✓

Worked example 3

The elevation of the top of a **60 m** tower is **60°**. Find the observer's distance.

Solve $\rightarrow d = h / \tan 60^\circ = 60 / \sqrt{3} = 20\sqrt{3} \approx \mathbf{34.64 \text{ m}}$. ✓



Bigger angle $\rightarrow$ nearer the object (steeper look-up). Smaller angle $\rightarrow$ farther away. At 30° the base is $\sqrt{3}$ times the height; at 60° the base is height/ $\sqrt{3}$.

Try these

$$d = 80, \theta = 30^\circ \rightarrow h = 80 / \sqrt{3} \approx \mathbf{46.19}$$

$$h = 40, \theta = 45^\circ \rightarrow d = \mathbf{40}$$

$$h = 90, \theta = 60^\circ \rightarrow d = 90 / \sqrt{3} = 30\sqrt{3} \approx \mathbf{51.96}$$

$$d = 50, \theta = 60^\circ \rightarrow h = 50\sqrt{3} \approx \mathbf{86.6}$$

④ Observer Moves – two angles

Walk a distance d toward a tower; the elevation grows from α to β . One height, two right triangles sharing it.

General result

- $h = d / (\cot\alpha - \cot\beta)$ (α is the far/smaller angle, β the near/larger)
- Special case $30^\circ \rightarrow 60^\circ$: $h = (\sqrt{3}/2) \cdot d$

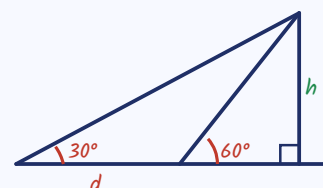
Derivation – $30^\circ \rightarrow 60^\circ$

At 60° (near, base = x): $\tan 60^\circ = h/x \Rightarrow x = h/\sqrt{3}$.

At 30° (far, base = $x+d$): $\tan 30^\circ = h/(x+d) \Rightarrow x+d = h\sqrt{3}$.

Subtract $\rightarrow d = h\sqrt{3} - h/\sqrt{3} = h(2/\sqrt{3})$.

$\therefore h = \sqrt{3} \cdot d / 2$. ✓



Worked example – $30^\circ \rightarrow 60^\circ$

A man walks **40 m** toward a tower; the elevation of the top rises from **30° to 60°** . Find the height.

Solve $\rightarrow h = (\sqrt{3}/2) \times 40 = 20\sqrt{3} \approx \mathbf{34.64 \text{ m}}$.

Check $\rightarrow$ via formula $h = 40 / (\cot 30^\circ - \cot 60^\circ) = 40 / (\sqrt{3} - 1/\sqrt{3}) = 40 / (2/\sqrt{3}) = 20\sqrt{3}$.



✓ Once you have h , the remaining distance to the base is $x = h/\sqrt{3} = 20\sqrt{3}/\sqrt{3} = 20 \text{ m}$ – exactly half of the 40 m walked.

④ Observer Moves – more cases

The formula $h = d / (\cot \alpha - \cot \beta)$ handles every angle pair – just plug the cot values.

Worked example – $30^\circ \rightarrow 45^\circ$

Walking **100 m** toward a tower changes the elevation from **30° to 45°** . Find the height.

Solve $\rightarrow h = 100 / (\cot 30^\circ - \cot 45^\circ) = 100 / (\sqrt{3} - 1)$.

Rationalise $\rightarrow \times(\sqrt{3}+1)/(\sqrt{3}+1)$: $h = 100(\sqrt{3}+1)/2 = 50(\sqrt{3}+1) \approx \mathbf{136.6 \text{ m}}$. 

Worked example – $45^\circ \rightarrow 60^\circ$

Walking **20 m** nearer, the elevation rises from **45° to 60°** . Find the height.

Solve $\rightarrow h = 20 / (\cot 45^\circ - \cot 60^\circ) = 20 / (1 - 1/\sqrt{3}) = 20\sqrt{3} / (\sqrt{3} - 1)$.

Rationalise $\rightarrow h = 20\sqrt{3}(\sqrt{3}+1)/2 = 10(3+\sqrt{3}) \approx \mathbf{47.32 \text{ m}}$. 



Moving *away* instead of toward? Same formula – the elevation now **drops** ($\beta \rightarrow \alpha$), and d is the distance walked back. Answer for the height is identical.

Worked example – find the distances too

Tower height from the $30^\circ \rightarrow 60^\circ$ case with $d = 40 \text{ m}$ was $h = 20\sqrt{3}$.

• Near base (60°): $x = h / \sqrt{3} = 20\sqrt{3} / \sqrt{3} = \mathbf{20 \text{ m}}$.

• Far base (30°): $x+d = h\sqrt{3} = 20\sqrt{3} \times \sqrt{3} = \mathbf{60 \text{ m}}$. ($60 - 20 = 40 \checkmark$)

Try these (find h)

$$d=60, 30^\circ \rightarrow 60^\circ \rightarrow h = 30\sqrt{3} \approx \mathbf{51.96}$$

$$d=50, 30^\circ \rightarrow 45^\circ \rightarrow 25(\sqrt{3}+1) \approx \mathbf{68.3}$$

$$d=90, 30^\circ \rightarrow 60^\circ \rightarrow h = 45\sqrt{3} \approx \mathbf{77.94}$$

$$d=10, 45^\circ \rightarrow 60^\circ \rightarrow 5(3+\sqrt{3}) \approx \mathbf{23.66}$$

⑤ Shadow Problems ☀ L

The sun's rays are the line of sight; the pole's **shadow is the base**. Lower the sun (smaller elevation) → longer the shadow.

Shadow facts (pole height h)

- shadow = $h / \tan\theta = h \cdot \cot\theta$
- $\theta = 45^\circ \rightarrow \text{shadow} = h$ • $\theta = 30^\circ \rightarrow \text{shadow} = h\sqrt{3}$ • $\theta = 60^\circ \rightarrow \text{shadow} = h/\sqrt{3}$

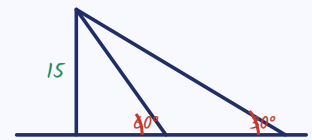
Worked example

The sun's elevation drops from **60° to 30°** . The shadow of a **15 m** pole lengthens — by how much?

At 60° → $s_1 = 15/\sqrt{3} = 5\sqrt{3} \approx 8.66$ m.

At 30° → $s_2 = 15\sqrt{3} \approx 25.98$ m.

Increase → $s_2 - s_1 = 15\sqrt{3} - 5\sqrt{3} = 10\sqrt{3} \approx \mathbf{17.32}$ m. ✓



Reverse question: "shadow = $\sqrt{3} \times \text{height} \rightarrow$ sun's elevation?" Since $\cot\theta = \sqrt{3}$, $\theta = 30^\circ$. If shadow = height → 45° .

Worked example — $45^\circ \rightarrow 30^\circ$

A **10 m** pole's shadow when the sun sinks from **45° to 30°** : increase = $10\sqrt{3} - 10 = 10(\sqrt{3}-1) \approx \mathbf{7.32}$ m. ✓

Try these

$h=20$, $\theta=30^\circ \rightarrow \text{shadow} = 20\sqrt{3} \approx \mathbf{34.64}$

shadow = $h \rightarrow \theta = \mathbf{45^\circ}$

$h=12$, $\theta=60^\circ \rightarrow \text{shadow} = 12/\sqrt{3} = 4\sqrt{3} \approx \mathbf{6.93}$

shadow = $h/\sqrt{3} \rightarrow \theta = \mathbf{60^\circ}$

⑥ Depression to Two Objects

From the top of a building of height h , look down at two objects on the ground — a **near** one (angle β) and a **far** one (angle α), $\beta > \alpha$.

Key result

- near distance = $h \cdot \cot \beta$ • far distance = $h \cdot \cot \alpha$
- gap between objects = $h(\cot \alpha - \cot \beta)$

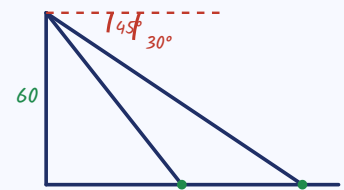
Worked example

From the top of a **60 m** building the depressions of two cars (same side, in a line) are **45°** (near) and **30°** (far). Find the distance between them.

Near $\rightarrow 60 \cdot \cot 45^\circ = 60$.

Far $\rightarrow 60 \cdot \cot 30^\circ = 60\sqrt{3} \approx 103.9$.

Gap $\rightarrow 60\sqrt{3} - 60 = 60(\sqrt{3} - 1) \approx \mathbf{43.9 \text{ m}}$. 



Always convert depression $\rightarrow$ elevation first (they are equal). Then it is just two single triangles sharing the same height h .

Worked example — find the height

From a cliff top the depressions of two boats **on opposite sides**, in line, are **45°** and **30°** ; the boats are **the sum** of the two feet apart. If the near boat is 40 m from the foot at 45° , then $h = 40 \cdot \tan 45^\circ = \mathbf{40 \text{ m}}$. Far boat = $40\sqrt{3} \approx 69.3 \text{ m}$. 

Try these (gap, same side)

$$h=30, 45^\circ \& 30^\circ \rightarrow 30(\sqrt{3}-1) \approx \mathbf{21.96}$$

$$h=50, 45^\circ \& 30^\circ \rightarrow 50(\sqrt{3}-1) \approx \mathbf{36.6}$$

$$h=90, 60^\circ \& 30^\circ \rightarrow 90(\sqrt{3}-1/\sqrt{3}) = 60\sqrt{3} \approx \mathbf{103.9}$$

$$h=45, 60^\circ \& 45^\circ \rightarrow 45(1-1/\sqrt{3}) \approx \mathbf{19.02}$$

⑦ Two Towers / Poles

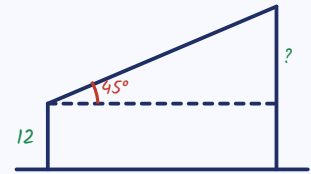
Two verticals on level ground a fixed distance apart. Sight the top of one from the foot or top of the other — drop a horizontal to make a right triangle.

Worked example 1 — top-to-top

A **12 m** pole and a taller tower stand **16 m** apart. From the top of the pole, the elevation of the tower's top is **45°**. Find the tower's height.

Rise above pole-top = $16 \cdot \tan 45^\circ = 16 \text{ m}$.

Tower = $12 + 16 = \mathbf{28 \text{ m}}$. ✓



Worked example 2 — elevation from the foot

Two towers stand **30 m** apart. From the **foot** of tower B the elevation of the top of tower A is **30°**. Find A's height.

Solve → $h_0 = 30 \cdot \tan 30^\circ = 30 / \sqrt{3} = 10\sqrt{3} \approx \mathbf{17.32 \text{ m}}$. ✓

Worked example 3 — up & down

From the top of a **20 m** building, the top of a tower has elevation **45°** and its foot has depression **45°**. Find the tower's height and the distance apart.

Depression 45° to the foot → horizontal distance = $20 \cdot \cot 45^\circ = 20 \text{ m}$.

Elevation 45° to top → extra height = $20 \cdot \tan 45^\circ = 20 \text{ m}$.

Tower = $20 + 20 = \mathbf{40 \text{ m}}$; distance apart = **20 m**. ✓



Trick → the horizontal gap is the **same** in both triangles. Compute it once, then use tan for each vertical piece.

Try these

10 m pole, gap 24, elev 45° top→top → **34 m**

gap 15, elev 60° from foot → $15\sqrt{3} \approx \mathbf{25.98}$

gap 40, elev 30° from foot → $40 / \sqrt{3} \approx \mathbf{23.09}$

15 m bldg, up 45° down 45° → tower **30 m**

⑧ Object Between Two Observers

A tower stands **between** two points on **opposite sides**, a distance d apart. Elevations are α and β . The two feet **add up** to d .

Key result (opposite sides)

$$h \cdot \cot \alpha + h \cdot \cot \beta = d \Rightarrow h = d / (\cot \alpha + \cot \beta)$$

(contrast: **same-side** / moving observer uses $\cot \alpha - \cot \beta$)

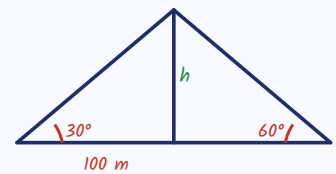
Worked example

A tower stands between two points **100 m** apart on opposite sides. The elevations of the top are **30°** and **60°** . Find the height.

Solve $\rightarrow h = 100 / (\cot 30^\circ + \cot 60^\circ) = 100 / (\sqrt{3} + 1/\sqrt{3})$.

$$\sqrt{3} + 1/\sqrt{3} = (3+1)/\sqrt{3} = 4/\sqrt{3}.$$

So $\rightarrow h = 100\sqrt{3}/4 = 25\sqrt{3} \approx \mathbf{43.3 \text{ m}}$. 



Opposite sides $\rightarrow$ **ADD** the cotangents; **same side** $\rightarrow$ **SUBTRACT**. One sign error flips the whole answer – picture which side each observer stands on.

Worked example – equal angles

Opposite points **80 m** apart, both elevations **45°** : $h = 80 / (\cot 45^\circ + \cot 45^\circ) = 80 / 2 = \mathbf{40 \text{ m}}$.

Tower sits exactly mid-way. 

Try these (opposite sides)

$$d=60, 30^\circ \& 60^\circ \rightarrow 15\sqrt{3} \approx \mathbf{25.98}$$

$$d=100, 45^\circ \& 45^\circ \rightarrow \mathbf{50}$$

$$d=120, 30^\circ \& 60^\circ \rightarrow 30\sqrt{3} \approx \mathbf{51.96}$$

$$d=40, 30^\circ \& 60^\circ \rightarrow 10\sqrt{3} \approx \mathbf{17.32}$$

⑨ Broken Tree & Ladder

The broken part / the ladder is the **hypotenuse**. Use $\sin$, $\cos$, $\tan$ on the leaning line.

Broken tree (breaks at height, top touches ground)

- standing part = $x \cdot \tan \theta$ • broken part = $x \cdot \sec \theta = x / \cos \theta$
- total height = $x(\tan \theta + \sec \theta)$ (x = distance of top from foot)

Worked example – broken tree

A tree breaks in the wind; the top touches the ground **10 m** from the foot, making **30°** with the ground. Find the original height.

Standing = $10 \cdot \tan 30^\circ = 10 / \sqrt{3} \approx 5.77$.

Broken = $10 \cdot \sec 30^\circ = 20 / \sqrt{3} \approx 11.55$.

Total = $30 / \sqrt{3} = 10\sqrt{3} \approx 17.32 \text{ m}$. ✓



Worked example – ladder

A ladder leans against a wall making **60°** with the ground; its foot is **2.5 m** from the wall. Find the ladder's length and the height reached.

Length = $\text{base} / \cos 60^\circ = 2.5 / 0.5 = 5 \text{ m}$.

Height = $\text{base} \cdot \tan 60^\circ = 2.5\sqrt{3} \approx 4.33 \text{ m}$. ✓



For a ladder: height reached = $\text{length} \cdot \sin \theta$, base = $\text{length} \cdot \cos \theta$. If length & angle are given, use $\sin/\cos$ directly – no need for the base first.

⑨ Observer of Given Height

When the person's **eye height** is given, the angle is measured from the **eye level**, not the ground. Solve for the part **above the eye**, then add the eye height.

Rule

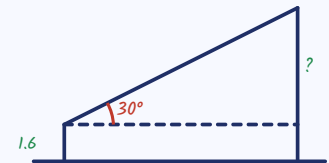
- height above eye = $d \cdot \tan \theta$
- tower height = $(d \cdot \tan \theta) + \text{eye height}$

Worked example 1

A man **1.6 m** tall stands **$20\sqrt{3}$ m** from a tower. The elevation of the top, from his eyes, is **30°** . Find the tower's height.

Above eye = $20\sqrt{3} \cdot \tan 30^\circ = 20\sqrt{3} / \sqrt{3} = 20$ m.

Tower = $20 + 1.6 = 21.6$ m. ✓



Worked example 2 – find the distance

A boy **1.5 m** tall sees the top of a **51.5 m** tower at elevation **45°** from his eyes. How far is he standing?

Above eye = $51.5 - 1.5 = 50$ m.

Distance = $50 / \tan 45^\circ = 50$ m. ✓



If the question ignores the observer's height, treat it as 0 – the angle is then measured from the ground. Read carefully whether "from his eyes" appears.

Worked example 3 – on a platform

From a **10 m** high platform the elevation of a tower top is **45°** and its base's depression is **30°** . Horizontal distance = $10 \cdot \cot 30^\circ = 10\sqrt{3} \approx 17.32$ m. Extra height = $10\sqrt{3} \cdot \tan 45^\circ = 10\sqrt{3}$.

Tower = $10 + 10\sqrt{3} \approx 27.32$ m. ✓

⑩ Flagstaff & River-Width

A flagstaff on a tower gives **two** elevations from one point — to the top of the tower and to the top of the flag.

From a point distance d away

- tower = $d \cdot \tan \alpha$ • tower + flag = $d \cdot \tan \beta$
- flag height = $d(\tan \beta - \tan \alpha)$

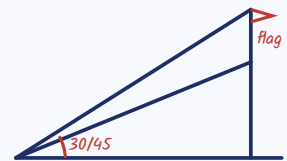
Worked example — flagstaff

From a point **30 m** from a tower's base, the top of the tower has elevation **30°** and the top of a flag on it, **45°** . Find the flag's height.

Tower = $30 \cdot \tan 30^\circ = 30 / \sqrt{3} = 10\sqrt{3} \approx 17.32$ m.

Tower+flag = $30 \cdot \tan 45^\circ = 30$ m.

Flag = $30 - 10\sqrt{3} \approx$ **12.68 m**. ✓



Worked example — width of a river

A tree stands on a river bank. From a point on the opposite bank directly across, the top has elevation **60°** . Moving **20 m** straight back, the elevation is **30°** . Find the river's width and tree height.

This is a $30^\circ \rightarrow 60^\circ$ move $\rightarrow$ tree $h = (\sqrt{3}/2) \times 20 = 10\sqrt{3} \approx 17.32$ m.

Width (base at 60°) = $h / \sqrt{3} = 10\sqrt{3} / \sqrt{3} =$ **10 m**; tree $\approx$ **17.32 m**. ✓



River-width is just the "observer moves away" case in disguise — the width is the **near** base.
Find the height first, then divide by $\tan$ of the near angle.

11 Quick Ratio Facts

Exam-winning shortcuts — memorise the ratios so you skip the full working.

Remember these

- $\theta = 45^\circ \Rightarrow \text{height} = \text{distance}$ (and shadow = height).
- $30^\circ \rightarrow 60^\circ$ walk d : $h = \sqrt{3}/2 \cdot d$, and the **remaining** distance = half of d .
- $\tan 30^\circ \cdot \tan 60^\circ = 1$ (so $\tan 30^\circ = \cot 60^\circ$).
- base at 30° : base at $60^\circ = \sqrt{3} : 1/\sqrt{3} = 3 : 1$ (for the same height).

Given (height h)	Base / shadow	Ratio base:h
$\theta = 30^\circ$	$h\sqrt{3}$	$\sqrt{3} : 1$
$\theta = 45^\circ$	h	$1 : 1$
$\theta = 60^\circ$	$h/\sqrt{3}$	$1 : \sqrt{3}$

Mini-example — use the ratio

Elevation 30° , tower 50 m $\rightarrow$ base = $50\sqrt{3} \approx 86.6$ m instantly (no tan needed). At 60° the same tower's base = $50/\sqrt{3} \approx \mathbf{28.87}$ m. 

Config cheat-sheet — which formula?

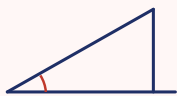
- Move toward / same side $\rightarrow h = d / (\cot \alpha - \cot \beta)$
- Object between, opposite sides $\rightarrow h = d / (\cot \alpha + \cot \beta)$
- Two depressions, same side $\rightarrow \text{gap} = h(\cot \alpha - \cot \beta)$
- Flagstaff (two elevations) $\rightarrow \text{flag} = d(\tan \beta - \tan \alpha)$



— for same side, + for opposite sides. Spot the picture, pick the sign, plug the cots — done in seconds.

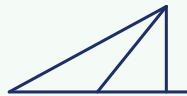
Standard Results at a Glance

Single triangle



$$h = d \cdot \tan \theta$$

Move toward



$$h = d / (\cot \alpha - \cot \beta)$$

Between (opp.)



$$h = d / (\cot \alpha + \cot \beta)$$

Every formula in one box

- $h = d \cdot \tan \theta$ • shadow = $h \cdot \cot \theta$ • depression = elevation
- toward: $h = d / (\cot \alpha - \cot \beta)$ • $30^\circ \rightarrow 60^\circ$: $h = \sqrt{3}d / 2$
- between (opp): $h = d / (\cot \alpha + \cot \beta)$
- two depressions: gap = $h(\cot \alpha - \cot \beta)$
- broken tree: total = $x(\tan \theta + \sec \theta)$ • flag: $d(\tan \beta - \tan \alpha)$

Warm-up worked – pick the right tool

Tower between two points 90 m apart, opposite sides, elevations 30° & 60° .

Opposite $\rightarrow$ add: $h = 90 / (\sqrt{3} + 1/\sqrt{3}) = 90\sqrt{3}/4 = 22.5\sqrt{3} \approx \mathbf{38.97 \text{ m.}}$ 

Try these (mixed)

tower 70, elev $45^\circ \rightarrow d = \mathbf{70}$

pole 24, sun $30^\circ \rightarrow$ shadow = $24\sqrt{3} \approx \mathbf{41.57}$

$d=80$ walk $30^\circ \rightarrow 60^\circ \rightarrow h = 40\sqrt{3} \approx \mathbf{69.3}$

flag: $d=40$, 30° & $45^\circ \rightarrow 40 - 40/\sqrt{3} \approx \mathbf{16.9}$

Solved Examples $\geq$

Q1 – single triangle

Tower 150 m, elevation 30° . Distance? $\rightarrow d = 150 \cdot \cot 30^\circ = 150\sqrt{3} \approx \mathbf{259.8 \text{ m}}$.

Q2 – observer moves

Walk 30 m, elevation $30^\circ \rightarrow 60^\circ$. Height? $\rightarrow h = \sqrt{3}/2 \times 30 = 15\sqrt{3} \approx \mathbf{25.98 \text{ m}}$.

Q3 – two depressions (same side)

Building 45 m, depressions 45° & 30° to two cars. Gap = $45(\cot 30^\circ - \cot 45^\circ) = 45(\sqrt{3} - 1) \approx \mathbf{32.9 \text{ m}}$.

Q4 – object between (opposite)

Points 60 m apart opposite sides, elevations 30° & 60° . $h = 60 / (\sqrt{3} + 1 / \sqrt{3}) = 60\sqrt{3} / 4 = 15\sqrt{3} \approx \mathbf{25.98 \text{ m}}$.

Q5 – flagstaff

Point 20 m away; top of tower 45° , top of flag 60° . Tower = $20 \cdot \tan 45^\circ = 20$;
tower+flag = $20\sqrt{3} \approx 34.64$. Flag = $20(\sqrt{3} - 1) \approx \mathbf{14.64 \text{ m}}$.



Q6 – shadow

Pole 10 m; sun's elevation $45^\circ \rightarrow 30^\circ$. Increase in shadow = $10\sqrt{3} - 10 = 10(\sqrt{3} - 1) \approx \mathbf{7.32 \text{ m}}$.



Golden habit $\rightarrow$ draw the triangle first, label h , d , θ , decide same-side ($-$) or opposite ($+$), then substitute. The diagram prevents 90% of errors.

Practice Set

1. Tower 50 m, elev 45° . Distance?
2. Tower 100 m, elev 30° . Distance?
3. Elev 60° , distance 30 m. Height?
4. Walk 50 m, $30^\circ \rightarrow 60^\circ$. Tower height?
5. Pole 20 m, sun 30° . Shadow?
6. Shadow = height. Sun's elevation?
7. Building 40 m, depr 30° to a car. Distance?
8. 10 m pole, gap 24 m, top $\rightarrow$ top 45° . Other?
9. Between, 80 m apart, 30° & 60° . Height?
10. Point 15 m away, elev 60° . Height?
11. Ladder 10 m at 60° . Height reached?
12. Flag: 10 m away, 30° tower, 45° flag. Flag?
13. $\tan 30^\circ \cdot \tan 60^\circ = ?$
14. Man 1.7 m, elev 45° , d 40 m. Tower?
15. Tree breaks, 8 m from foot at 30° . Height?

Answer Key – with working

$1 \rightarrow 50 \text{ m } (\tan 45^\circ = 1)$ · $2 \rightarrow 100\sqrt{3} \approx 173.2$ ($d = 100 \cdot \cot 30^\circ$) · $3 \rightarrow 30\sqrt{3} \approx 51.96$ ($h = 30 \cdot \tan 60^\circ$) · $4 \rightarrow 25\sqrt{3} \approx 43.3$ ($\sqrt{3}/2 \times 50$) · $5 \rightarrow 20\sqrt{3} \approx 34.64$ ($20 \cdot \cot 30^\circ$) · $6 \rightarrow 45^\circ$ ($\cot \theta = 1$) · $7 \rightarrow 40\sqrt{3} \approx 69.28$ ($40 \cdot \cot 30^\circ$) · $8 \rightarrow 34 \text{ m } (10 + 24 \cdot \tan 45^\circ)$ · $9 \rightarrow 20\sqrt{3} \approx 34.64$ ($80\sqrt{3}/4$) · $10 \rightarrow 15\sqrt{3} \approx 25.98$ ($15 \cdot \tan 60^\circ$) · $11 \rightarrow 5\sqrt{3} \approx 8.66$ ($10 \cdot \sin 60^\circ$) · $12 \rightarrow 10 - 10/\sqrt{3} \approx 4.23$ ($10 - 10 \cdot \tan 30^\circ$) · $13 \rightarrow 1$ ($(1/\sqrt{3}) \cdot \sqrt{3}$) · $14 \rightarrow 41.7 \text{ m } (40 \cdot \tan 45^\circ + 1.7)$ · $15 \rightarrow 8\sqrt{3} \approx 13.86$ ($8(\tan 30^\circ + \sec 30^\circ)$).

Quick Revision

- ✓ $\tan \theta = \text{height} / \text{base}$. $\tan 30^\circ = 1/\sqrt{3}$, $\tan 45^\circ = 1$, $\tan 60^\circ = \sqrt{3}$; $\sqrt{3} \approx 1.732$.
- ✓ Depression = elevation. Move toward $\rightarrow$ subtract cots; between (opp.) $\rightarrow$ add cots.
- ✓ $45^\circ \Rightarrow \text{height} = \text{distance}$. $30^\circ \rightarrow 60^\circ$ walk $d \Rightarrow h = \sqrt{3}d/2$, remaining = $d/2$.

☆ Look up, look down – solve with tan! ☆