

Mensuration



~ Quantitative Aptitude · SSC Exams ~

← green boxes = formulas · navy boxes = worked sums!



Mensuration = measuring shapes. **2D figures** → area & perimeter. **3D solids** → surface area & volume.

Perimeter → length of the boundary (units).

Area → region enclosed (unit^2).

CSA / LSA → curved / lateral surface.

TSA → total surface (unit^2).

Volume → space filled (unit^3).

Take $\pi = 22/7$ or 3.14 .

① Square & Rectangle

SQUARE – side = a

- Area = a^2 ex: $a=6 \rightarrow \text{area} = 36$
- Perimeter = $4a$ ex: $a=6 \rightarrow 4 \times 6 = 24$
- Diagonal = $a\sqrt{2}$ ex: $a=6 \rightarrow 6\sqrt{2} \approx 8.49$

RECTANGLE – length l , breadth b

- Area = $l \times b$ ex: $8 \times 6 = 48$
- Perimeter = $2(l + b)$ ex: $2(8+6) = 28$
- Diagonal = $\sqrt{l^2 + b^2}$ ex: $\sqrt{64+36} = \sqrt{100} = 10$

Worked example – rectangular field $40 \text{ m} \times 30 \text{ m}$

Area = $40 \times 30 = 1200 \text{ m}^2$.

Perimeter = $2(40 + 30) = 140 \text{ m}$.

Diagonal = $\sqrt{40^2 + 30^2} = \sqrt{2500} = 50 \text{ m}$.

Fencing cost at ₹25/m = $140 \times 25 = ₹3500$. 

② Triangle – Area & Heron

- Area = $\frac{1}{2} \times \text{base} \times \text{height}$ ex: $b=10, h=6 \rightarrow \frac{1}{2} \times 10 \times 6 = 30$
- Right triangle Area = $\frac{1}{2} \times \text{base} \times \text{perpendicular}$ ex: legs 6, 8 $\rightarrow \frac{1}{2} \times 6 \times 8 = 24$
- Hypotenuse = $\sqrt{(\text{base}^2 + \text{perp}^2)}$ ex: $\sqrt{(36+64)} = 10$

HERON'S FORMULA – sides a, b, c

- Semi-perimeter $s = (a + b + c) / 2$
 - Area = $\sqrt{s(s-a)(s-b)(s-c)}$
- ex: sides 3, 4, 5 $\rightarrow s=6, \text{area}=\sqrt{(6 \cdot 3 \cdot 2 \cdot 1)}=\sqrt{36} = 6$

Worked example – triangle with sides 13, 14, 15

Step 1 – $s = (13+14+15) / 2 = 42 / 2 = 21$.

Step 2 – Area = $\sqrt{(21 \cdot (21-13) \cdot (21-14) \cdot (21-15))} = \sqrt{(21 \cdot 8 \cdot 7 \cdot 6)}$.

Step 3 – $= \sqrt{7056} = 84 \text{ unit}^2$.

Height on base 14 $\rightarrow \frac{1}{2} \times 14 \times h = 84 \therefore h = 12$. 



Use Heron only when the height is unknown. If you know base & height, plain $\frac{1}{2}bh$ is faster.

Special triangle facts

- **Isosceles** (equal sides a , base b): Area = $(b/4)\sqrt{(4a^2-b^2)}$.
- Common Pythagorean triples: (3,4,5) (5,12,13) (8,15,17) (7,24,25).
- Area = $\frac{1}{2} \times d_1 \times d_2$ when the two known sides meet at 90° .

Mini – isosceles, equal sides 10, base 12

Area = $(12/4)\sqrt{(4 \cdot 100 - 144)} = 3\sqrt{256} = 3 \times 16 = 48 \text{ unit}^2$.

③ Equilateral Triangle

Side = a (all sides & angles equal)

- Area = $(\sqrt{3}/4) a^2$ ex: $a=4 \rightarrow (\sqrt{3}/4) \times 16 = 4\sqrt{3} \approx 6.93$
- Height = $(\sqrt{3}/2) a$ ex: $a=4 \rightarrow 2\sqrt{3} \approx 3.46$
- Perimeter = $3a$ ex: $a=4 \rightarrow 12$

Worked example – equilateral side 6

Area = $(\sqrt{3}/4) \times 6^2 = (\sqrt{3}/4) \times 36 = 9\sqrt{3} \approx 15.59$.

Height = $(\sqrt{3}/2) \times 6 = 3\sqrt{3} \approx 5.20$.

Perimeter = $3 \times 6 = 18$. 

④ Parallelogram

- Area = **base $\times$ height** ex: base=12, h=5 $\rightarrow 60$
- Perimeter = $2(a + b)$, a, b = adjacent sides ex: $2(12+8) = 40$
- Area (diagonals d_1, d_2 , angle θ) = $\frac{1}{2} d_1 d_2 \sin\theta$

Worked example – base 15, height 8

Area = base $\times$ height = $15 \times 8 = 120 \text{ unit}^2$.

If adjacent side = 10 $\rightarrow$ Perimeter = $2(15+10) = 50$.

Note: the height is the $\perp$ distance between the two parallel bases, not the slant side. 

 A diagonal splits a parallelogram into two equal triangles $\therefore$ each triangle = $\frac{1}{2} \times$ area of the parallelogram.

⑤ Rhombus

Diagonals d_1, d_2 (they bisect at 90°)

- Area = $\frac{1}{2} \times d_1 \times d_2$ ex: $16 \times 12 \rightarrow \frac{1}{2} \times 16 \times 12 = 96$
- Side = $\sqrt{(d_1/2)^2 + (d_2/2)^2}$ ex: $\sqrt{8^2 + 6^2} = 10$
- Perimeter = $4 \times \text{side}$ ex: $4 \times 10 = 40$

Worked example – rhombus, diagonals 24 & 10

Area = $\frac{1}{2} \times 24 \times 10 = 120 \text{ unit}^2$.

Side = $\sqrt{12^2 + 5^2} = \sqrt{169} = 13$.

Perimeter = $4 \times 13 = 52$. 

⑥ Trapezium

- Area = $\frac{1}{2} \times (\text{sum of parallel sides}) \times \text{height}$
= $\frac{1}{2} \times (a + b) \times h$ ex: $a=10, b=14, h=6 \rightarrow \frac{1}{2} \times 24 \times 6 = 72$
- Height (h) = the $\perp$ gap between the two parallel sides.

Worked example – parallel sides 20 & 12, height 8

Area = $\frac{1}{2} \times (20 + 12) \times 8 = \frac{1}{2} \times 32 \times 8 = 128 \text{ unit}^2$.

Reverse $\rightarrow$ if area 128 & sides 20, 12 known, then $h = 2 \times 128 / 32 = 8$. 



Memory hook $\rightarrow$ trapezium area = average of parallel sides $\times$ height. Only two sides are parallel (else it is a parallelogram).

⑦ Circle & Semicircle

CIRCLE – radius r (diameter = $2r$)

- Area = πr^2 ex: $r=7 \rightarrow (22/7) \times 49 = 154$
- Circumference = $2\pi r$ ex: $r=7 \rightarrow 2 \times (22/7) \times 7 = 44$

SEMICIRCLE – half a circle

- Area = $\frac{1}{2} \pi r^2$ ex: $r=7 \rightarrow \frac{1}{2} \times 154 = 77$
- Perimeter = $\pi r + 2r$ ex: $r=7 \rightarrow 22 + 14 = 36$
(perimeter = curved arc πr plus the straight diameter $2r$)

Worked example – circle of radius 14 cm ($\pi = 22/7$)

Area = $(22/7) \times 14^2 = (22/7) \times 196 = 616 \text{ cm}^2$.

Circumference = $2 \times (22/7) \times 14 = 88 \text{ cm}$.

Diameter = $2 \times 14 = 28 \text{ cm}$.

Semicircle area = $\frac{1}{2} \times 616 = 308 \text{ cm}^2$. 

✓ Pick $r = \text{a multiple of } 7$ and $\pi = 22/7$ – the 7's cancel and the answer comes out clean. Exam-setters love $r = 7, 14, 21$.

Handy circle relations

- Area in terms of diameter: $\pi d^2 / 4$. • Area in terms of circumference C : $C^2 / 4\pi$.
- If radius $\times 2 \rightarrow$ area $\times 4$ (area scales with r^2).

⑧ Sector of a Circle

Angle θ (degrees), radius r

- Area = $(\theta/360^\circ) \times \pi r^2$ ex: $\theta=90^\circ, r=14 \rightarrow \frac{1}{4} \times 616 = 154$
- Arc length = $(\theta/360^\circ) \times 2\pi r$ ex: $\theta=90^\circ, r=14 \rightarrow \frac{1}{4} \times 88 = 22$
- Perimeter of sector = arc + $2r$ ex: $22 + 28 = 50$

Worked example – $\theta = 60^\circ, r = 21 \text{ cm}$

Area = $(60/360) \times (22/7) \times 21^2 = \frac{1}{6} \times (22/7) \times 441 = \frac{1}{6} \times 1386 = 231 \text{ cm}^2$.

Arc = $(60/360) \times 2 \times (22/7) \times 21 = \frac{1}{6} \times 132 = 22 \text{ cm}$. 

⑨ Ring / Annulus

Outer radius R , inner radius r

• Area of ring = $\pi(R^2 - r^2) = \pi(R+r)(R-r)$

ex: $R=14, r=7 \rightarrow (22/7)(196-49) = (22/7) \times 147 = 462$

Worked example – circular track, $R = 21 \text{ m}, r = 14 \text{ m}$

Area of path = $(22/7)(21^2 - 14^2) = (22/7)(441 - 196) = (22/7) \times 245 = 770 \text{ m}^2$.

Width of the path = $R - r = 21 - 14 = 7 \text{ m}$. 



A path/border around a circle is a ring. A path inside a rectangle $\rightarrow$ big rectangle area – inner rectangle area.

2D Formula Recap – at a glance

Shape	Area	Perimeter / other
Square	a^2	$4a \cdot \text{diag} = a\sqrt{2}$
Rectangle	$l \times b$	$2(l+b) \cdot \text{diag} = \sqrt{l^2+b^2}$
Triangle	$\frac{1}{2} \times b \times h$	Heron $\sqrt{s(s-a)(s-b)(s-c)}$
Equilateral	$(\sqrt{3}/4) a^2$	$3a \cdot \text{height} = (\sqrt{3}/2) a$
Parallelogram	base $\times$ height	$2(a+b)$
Rhombus	$\frac{1}{2} d_1 d_2$	$4 \times \text{side} \cdot \text{side} = \sqrt{(d_1/2)^2 + (d_2/2)^2}$
Trapezium	$\frac{1}{2}(a+b) \times h$	sum of all 4 sides
Circle	πr^2	$2\pi r$
Semicircle	$\frac{1}{2} \pi r^2$	$\pi r + 2r$
Sector	$(\theta/360^\circ) \pi r^2$	$\text{arc} = (\theta/360^\circ) 2\pi r$
Ring	$\pi(R^2 - r^2)$	outer $2\pi R$, inner $2\pi r$

Mixed worked – a square & a circle have equal perimeters

Square side = 11 $\rightarrow$ perimeter = 44. Circle circumference = 44 = $2\pi r \rightarrow 2 \times (22/7) \times r = 44 \therefore r = 7$.

Areas: square = $11^2 = 121$; circle = $(22/7) \times 49 = 154$. $\therefore$ the **circle encloses more area** for the same boundary. 

Mixed worked – path around a garden

A rectangular garden 20 m $\times$ 14 m has a 2 m wide path **outside** it. Outer size = 24 $\times$ 18 = 432 m^2 ; garden = 280 m^2 .

Area of path = 432 – 280 = **152 m^2** . 

 Golden 2D rule $\rightarrow$ area is always in **square** units; if lengths double, area becomes 4 $\times$ (scales as the square of the ratio).



PART B — 3D SOLIDS. Now shapes have depth: measure **Volume** (space filled, unit³) and **Surface Area** — CSA/LSA (curved / lateral) and **TSA** (total).

⑩ Cube

Edge = a

- Volume = a^3 ex: $a=5 \rightarrow 125$ • Diagonal = $a\sqrt{3}$ ex: $5\sqrt{3} \approx 8.66$
- TSA = $6a^2$ ex: $6 \times 25 = 150$ • LSA = $4a^2$ ex: $4 \times 25 = 100$

Worked — cube of edge 6 cm

Volume = $6^3 = 216 \text{ cm}^3$ • TSA = $6 \times 36 = 216 \text{ cm}^2$ • LSA = $4 \times 36 = 144$ • diagonal = $6\sqrt{3} \approx 10.39 \text{ cm}$. 

⑪ Cuboid

Length l , breadth b , height h

- Volume = $l \times b \times h$ ex: $5 \times 4 \times 3 = 60$
- TSA = $2(lb + bh + hl)$ ex: $2(20 + 12 + 15) = 94$
- LSA = $2h(l + b)$ ex: $2 \times 3 \times 9 = 54$ • Diagonal = $\sqrt{l^2 + b^2 + h^2}$

Worked — room 6 m × 5 m × 4 m

Volume = $6 \times 5 \times 4 = 120 \text{ m}^3$ • TSA = $2(30 + 20 + 24) = 148 \text{ m}^2$ • LSA (4 walls) = $2 \times 4 \times 11 = 88 \text{ m}^2$
• diagonal = $\sqrt{36 + 25 + 16} = \sqrt{77} \approx 8.77 \text{ m}$. 



A cube is a cuboid with $l = b = h$. To paint 4 walls use LSA; to include floor+ceiling use TSA.

12 Cylinder

Radius r , height h

- Volume = $\pi r^2 h$ ex: $r=7, h=10 \rightarrow (22/7) \times 49 \times 10 = 1540$
- CSA = $2\pi rh$ ex: $2 \times (22/7) \times 7 \times 10 = 440$
- TSA = $2\pi r(r + h)$ ex: $2 \times (22/7) \times 7 \times 17 = 748$
- Hollow (pipe) volume = $\pi h(R^2 - r^2)$ R outer, r inner

Worked example – cylinder $r = 7$ cm, $h = 20$ cm

Volume = $(22/7) \times 7^2 \times 20 = (22/7) \times 49 \times 20 = 3080 \text{ cm}^3$.

CSA = $2 \times (22/7) \times 7 \times 20 = 880 \text{ cm}^2$.

TSA = $2 \times (22/7) \times 7 \times (7+20) = 44 \times 27 = 1188 \text{ cm}^2$. 

Hollow pipe – $R = 5$ cm, $r = 3$ cm, length 14 cm

Volume of metal = $\pi h(R^2 - r^2) = (22/7) \times 14 \times (25 - 9) = 44 \times 16 = 704 \text{ cm}^3$. 

✓ TSA = CSA + two circular ends ($2\pi r^2$). An open tank/tin (no lid) has area = CSA + one base = $2\pi rh + \pi r^2$.

Quick check

$r=14, h=5 \rightarrow V = (22/7) \cdot 196 \cdot 5 = 3080$

$r=3.5, h=10 \rightarrow V = (22/7) \cdot 12.25 \cdot 10 = 385$

$r=7, h=1 \rightarrow CSA = 2 \cdot 22 = 44$

1 L water fills how tall in $r=7$ tin? ≈ 6.5 cm

13 Cone

Radius r , height h , slant l

- Slant $l = \sqrt{r^2 + h^2}$ ex: $r=5, h=12 \rightarrow \sqrt{169} = 13$
- Volume $= \frac{1}{3} \pi r^2 h$ = $\frac{1}{3}$ of a cylinder of same r, h
- CSA $= \pi r l$ • TSA $= \pi r(r + l) = \text{CSA} + \text{base } \pi r^2$

Worked example – cone $r = 7 \text{ cm}, h = 24 \text{ cm}$

Slant $l = \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25 \text{ cm}$.

Volume $= \frac{1}{3} \times (22/7) \times 49 \times 24 = \frac{1}{3} \times 22 \times 7 \times 24 = 1232 \text{ cm}^3$.

CSA $= (22/7) \times 7 \times 25 = 550 \text{ cm}^2$.

TSA $= (22/7) \times 7 \times (7 + 25) = 22 \times 32 = 704 \text{ cm}^2$. 



l, r, h form a right triangle $\rightarrow l^2 = r^2 + h^2$. Watch out: CSA uses slant l , volume uses vertical height h – never mix them.

Cone vs Cylinder – same r & h

- Volume of cone $= \frac{1}{3} \times$ volume of cylinder (same base & height).
- $\therefore$ 3 cones of ice fill exactly 1 cylinder of the same r and h .
- Ex: cylinder $V = 1848 \rightarrow$ matching cone $V = 1848 / 3 = 616$.

Mini – find h from volume

Cone volume $462 \text{ cm}^3, r = 7 \rightarrow \frac{1}{3} \times (22/7) \times 49 \times h = 462 \rightarrow (154/3)h = 462 \therefore h = 9 \text{ cm}$.



14 Sphere

Radius r

- Volume = $\frac{4}{3} \pi r^3$ ex: $r=7 \rightarrow (\frac{4}{3}) \times (22/7) \times 343 \approx 1437.3$
- Surface area = $4\pi r^2$ ex: $r=7 \rightarrow 4 \times (22/7) \times 49 = 616$

15 Hemisphere

Radius r (half a sphere)

- Volume = $\frac{2}{3} \pi r^3$ ex: $r=7 \rightarrow (\frac{2}{3}) \times (22/7) \times 343 \approx 718.7$
- CSA (curved) = $2\pi r^2$ ex: $r=7 \rightarrow 2 \times 154 = 308$
- TSA = $3\pi r^2$ ex: $r=7 \rightarrow 3 \times 154 = 462$ (curved + flat circle)

Worked example – sphere of radius 21 cm

Surface area = $4 \times (22/7) \times 21^2 = 4 \times (22/7) \times 441 = 4 \times 1386 = 5544 \text{ cm}^2$.

Volume = $\frac{4}{3} \times (22/7) \times 21^3 = \frac{4}{3} \times (22/7) \times 9261 = \frac{4}{3} \times 29106 = 38808 \text{ cm}^3$. 

Worked example – hemispherical bowl, $r = 7 \text{ cm}$

CSA = $2 \times (22/7) \times 49 = 308 \text{ cm}^2$ • TSA = $3 \times 154 = 462 \text{ cm}^2$.

Capacity = $\frac{2}{3} \times (22/7) \times 343 = 718.67 \text{ cm}^3 \approx 0.72 \text{ litre}$. 



Sphere surface = $4\pi r^2$ = exactly 4 great circles. Hemisphere TSA has a $+\pi r^2$ for the flat top, so it is $3\pi r^2$, not $2\pi r^2$.

16 Frustum of a Cone

A frustum = a cone with its top sliced off parallel to the base. Outer (bottom) radius R , top radius r , vertical height h .

- Volume = $\frac{1}{3} \pi h (R^2 + Rr + r^2)$
- Slant $l = \sqrt{h^2 + (R - r)^2}$
- CSA = $\pi(R + r) l$ • TSA = CSA + $\pi R^2 + \pi r^2$ (both ends)

Worked example – bucket, $R = 6$, $r = 3$, $h = 4$ (cm)

$$R^2 + Rr + r^2 = 36 + 18 + 9 = 63.$$

$$\text{Volume} = \frac{1}{3} \times (22/7) \times 4 \times 63 = \frac{1}{3} \times (22/7) \times 252 = \frac{1}{3} \times 792 = 264 \text{ cm}^3.$$

$$\text{Slant } l = \sqrt{4^2 + (6-3)^2} = \sqrt{16+9} = \sqrt{25} = 5 \text{ cm}.$$

$$\text{CSA} = (22/7) \times (6+3) \times 5 = (22/7) \times 45 = \approx 141.4 \text{ cm}^2. \quad \checkmark$$

✓ Everyday frustums → a bucket, a drinking glass, a lampshade. If $r = 0$ the frustum becomes a full cone; if $R = r$ it becomes a cylinder.

Second worked – capacity in litres

A bucket has $R = 21$ cm, $r = 7$ cm, $h = 24$ cm. $R^2 + Rr + r^2 = 441 + 147 + 49 = 637$.

$$\text{Volume} = \frac{1}{3} \times (22/7) \times 24 \times 637 = \frac{1}{3} \times (22/7) \times 15288 = \frac{1}{3} \times 48048 = 16016 \text{ cm}^3.$$

$$\therefore \text{capacity} = 16016 \div 1000 = 16.016 \text{ litres.} \quad \checkmark$$



For CSA/TSA of a frustum use slant l ; for volume use vertical h . The two differ whenever $R \neq r$.

17 Prism

Uniform cross-section, length h

- Volume = (base area) $\times$ height
- LSA = (base perimeter) $\times$ height • TSA = LSA + 2 $\times$ base area

Worked – triangular prism, base 6-8-10, length 15

Base area = $\frac{1}{2} \times 6 \times 8 = 24 \rightarrow$ Volume = $24 \times 15 = 360$. Base perimeter = $6 + 8 + 10 = 24 \rightarrow$ LSA = $24 \times 15 = 360$; TSA = $360 + 2 \times 24 = 408$. 

18 Pyramid

Base area B , height h , slant height L

- Volume = $\frac{1}{3} \times$ (base area) $\times$ height
- LSA = $\frac{1}{2} \times$ (base perimeter) $\times$ slant height • TSA = LSA + base area

Worked – square pyramid, base side 10, height 12

Slant $L = \sqrt{h^2 + (\text{side}/2)^2} = \sqrt{12^2 + 5^2} = \sqrt{169} = 13$. Volume = $\frac{1}{3} \times 10^2 \times 12 = 400$; LSA = $\frac{1}{2} \times 40 \times 13 = 260$; TSA = $260 + 100 = 360$. 

3D Formula Recap

Solid	Volume	CSA / LSA	TSA
Cube	a^3	$4a^2$	$6a^2$
Cuboid	$l b h$	$2h(l+b)$	$2(lb+bh+hl)$
Cylinder	$\pi r^2 h$	$2\pi r h$	$2\pi r(r+h)$
Cone	$\frac{1}{3} \pi r^2 h$	$\pi r l$	$\pi r(r+l)$
Sphere	$\frac{4}{3} \pi r^3$	—	$4\pi r^2$
Hemisphere	$\frac{2}{3} \pi r^3$	$2\pi r^2$	$3\pi r^2$
Frustum	$\frac{1}{3} \pi h(R^2 + Rr + r^2)$	$\pi(R+r)l$	$CSA + \pi R^2 + \pi r^2$

Recasting, Melting & Units

Key facts to lock in

- Melting / recasting keeps **VOLUME constant** → equate the two volumes.
- $1 \text{ m}^3 = 1000 \text{ litres}$ · $1 \text{ litre} = 1000 \text{ cm}^3$ · $1 \text{ m} = 100 \text{ cm}$.
- Painting / plastering cost uses **surface area**; filling / capacity uses **volume**.

Worked 1 – sphere melted into a cylinder

Sphere $r = 6 \text{ cm}$ is melted & recast into a cylinder of base radius 4 cm . Find its height.

Equate volumes → $\frac{4}{3} \pi (6)^3 = \pi (4)^2 h \rightarrow \frac{4}{3} \times 216 = 16h \rightarrow 288 = 16h \therefore h = 18 \text{ cm}$. 

Worked 2 – cone recast into a sphere

Metallic cone $r = 6$, $h = 24$ is melted into a single sphere. Find the sphere's radius.

Cone $V = \frac{1}{3} \pi (36)(24) = 288\pi$. Sphere $\frac{4}{3} \pi R^3 = 288\pi \rightarrow R^3 = 216 \therefore R = 6 \text{ cm}$. 

Worked 3 – small cubes into one big cube

Three metal cubes of edges $3, 4, 5 \text{ cm}$ are melted into one cube. Edge = ?

Total volume = $27 + 64 + 125 = 216 \rightarrow \text{edge} = \sqrt[3]{216} = 6 \text{ cm}$. New TSA = $6 \times 6^2 = 216 \text{ cm}^2$.



Worked 4 – tank capacity & painting cost

A tank $2 \text{ m} \times 1.5 \text{ m} \times 1 \text{ m}$: capacity = $3 \text{ m}^3 = 3000 \text{ litres}$.

Paint its 4 walls: LSA = $2 \times 1 \times (2 + 1.5) = 7 \text{ m}^2$; at $\text{₹}40/\text{m}^2 \rightarrow \text{cost} = \text{₹}280$. 



In every recasting sum the π cancels – so never plug in $22/7$; just equate volumes and solve for the unknown.

Solved Examples – exam style Σ

Q1 – area of an equilateral triangle of side 12 cm

$$\text{Area} = (\sqrt{3}/4) \times 12^2 = (\sqrt{3}/4) \times 144 = 36\sqrt{3} = 36\sqrt{3} \approx 62.35 \text{ cm}^2. \quad \checkmark$$

Q2 – a wire is bent into a square of side 11, then reshaped into a circle

$$\text{Wire length} = 4 \times 11 = 44 = \text{circumference} = 2\pi r \rightarrow 2 \times (22/7) \times r = 44 \therefore r = 7.$$

$$\text{Circle area} = (22/7) \times 49 = 154 > \text{square area } 121 \rightarrow \text{the circle holds more area.} \quad \checkmark$$

Q3 – largest sphere carved from a cube of edge 14 cm

$$\text{Sphere diameter} = \text{cube edge} = 14 \rightarrow r = 7. \text{ Volume} = 4/3 \times (22/7) \times 343 = \approx 1437.3 \text{ cm}^3. \quad \checkmark$$

Q4 – cylinder has CSA 880 cm^2 and height 20 cm; find its volume

$$\text{CSA} = 2\pi rh \rightarrow 2 \times (22/7) \times r \times 20 = 880 \rightarrow r = 7. \text{ Volume} = (22/7) \times 49 \times 20 = 3080 \text{ cm}^3. \quad \checkmark$$

Q5 – ratio of volumes: cone : sphere : cylinder (same r, height = 2r)

$$\text{Cone } 1/3 \pi r^2(2r) = 2/3 \pi r^3; \text{ sphere } 4/3 \pi r^3; \text{ cylinder } \pi r^2(2r) = 2\pi r^3.$$

$$\text{Ratio} = 2/3 : 4/3 : 2 = 1 : 2 : 3. \quad \checkmark$$

Q6 – longest rod that fits in a room $12 \text{ m} \times 9 \text{ m} \times 8 \text{ m}$

$$\text{Longest rod} = \text{space diagonal} = \sqrt{12^2 + 9^2 + 8^2} = \sqrt{144 + 81 + 64} = \sqrt{289} = 17 \text{ m.} \quad \checkmark$$



Remember the trio ratio cone : sphere : cylinder = 1 : 2 : 3 (equal r, h = 2r) – a favourite one-line SSC question.

Practice Set

1. Area of square, side 15 cm?
2. Diagonal of rectangle 9×12 ?
3. Heron area of triangle 7, 24, 25?
4. Area of equilateral, side 10?
5. Area of rhombus, diagonals 30, 16?
6. Trapezium sides 8, 12, height 5?
7. Area of circle, $r = 21$ cm?
8. Ring area, $R = 10$, $r = 6$?
9. Volume of cube, edge 9?
10. Diagonal of cuboid 3, 4, 12?
11. Volume of cylinder $r=7$, $h=15$?
12. TSA of cone $r=7$, $l=25$?
13. Surface area of sphere $r=14$?
14. TSA of hemisphere $r=7$?
15. Sphere $r=3$ melted $\rightarrow$ wire? $V=?$

Answer Key – with working

$1 \rightarrow 225$ (15^2) · $2 \rightarrow 15$ ($\sqrt{81+144}$) · $3 \rightarrow 84$ ($\frac{1}{2} \times 7 \times 24$, right $\angle$) · $4 \rightarrow 25\sqrt{3}$ ($\sqrt{3}/4 \times 100$) · $5 \rightarrow 240$ ($\frac{1}{2} \times 30 \times 16$) · $6 \rightarrow 50$ ($\frac{1}{2} \times 20 \times 5$) · $7 \rightarrow 1386$ ($((22/7) \times 441)$) · $8 \rightarrow 64\pi \approx 201$ ($\pi(100-36)$) · $9 \rightarrow 729$ (9^3) · $10 \rightarrow 13$ ($\sqrt{9+16+144}$) · $11 \rightarrow 2310$ ($((22/7) \times 49 \times 15)$) · $12 \rightarrow 704$ ($\pi \times 7 \times 32$) · $13 \rightarrow 2464$ ($4 \times ((22/7) \times 196)$) · $14 \rightarrow 462$ (3×154) · $15 \rightarrow 36\pi \approx 113.1$ ($4/3 \pi \times 27$).

Quick Revision

- ✓ Square a^2 · Rect $l \times b$ · Triangle $\frac{1}{2}bh$ / Heron · Equilateral $(\sqrt{3}/4)a^2$ · Rhombus $\frac{1}{2}d_1d_2$ · Trapezium $\frac{1}{2}(a+b)h$ · Circle πr^2 .
- ✓ Cube a^3 , $6a^2$ · Cuboid $l b h$, $2(lb+bh+hl)$ · Cylinder $\pi r^2 h$, $2\pi r h$ · Cone $1/3 \pi r^2 h$, $\pi r l$ · Sphere $4/3 \pi r^3$, $4\pi r^2$.
- ✓ Recasting $\rightarrow$ equate volumes · $1 \text{ m}^3 = 1000 \text{ L}$ · paint $\rightarrow$ surface area · use $\pi = 22/7$.

☆ Area, surface, volume – formulas at your fingertips! ☆