Ratio and Proportion

~ Quantitative Aptitude · SSC Exams ~

← circled ①②③ = key sections!

A ratio compares two like quantities by division; a **proportion** says two ratios are equal. Master these and mixtures, partnership, ages & speed all fall into place.

① Ratio — the Basics %

The ratio of **a** to **b** is written **$a : b = a/b$** $(b \neq 0)$.

- **a** = **antecedent** (first term) · **b** = **consequent** (second term).
- A ratio is a pure number — it has **no units**; both quantities must be in the **same unit** first.

Example — put in the same unit first

20 minutes : 1 hour → write the hour as 60 min → 20 : 60 = **1 : 3**.

Never compare rupees with paise, or metres with cm, until the units match.

Simplest form

Divide both terms by their **HCF** to reach lowest terms. 84 : 105 → ÷21 → **4 : 5**

Golden property of a ratio

- A ratio is **unchanged** if both terms are × or ÷ by the **same non-zero number**.

$2 : 3 = 4 : 6 = 6 : 9 = 20 : 30$ (all equal to $2/3$)

- $a : b$ is a ratio of **greater inequality** if $a > b$, of **less inequality** if $a < b$.

*If $a : b = 3 : 4$, then the two numbers are **3k and 4k** for some k. Introducing this **k** is the single most useful trick of the chapter.*

Comparing & Manipulating Ratios

To decide which ratio is bigger, **cross-multiply** or bring both to a **common denominator**.

Worked example — which is greater, 3 : 4 or 5 : 7 ?

Cross-multiply → compare 3×7 with 4×5.

$3 \times 7 = \mathbf{21}$ and $4 \times 5 = \mathbf{20}$. Since $21 > 20$ →

Answer → **3 : 4 > 5 : 7**. ✅

Handy sub-rules (for a : b, all terms positive)

- Add same k to both terms → ratio of greater inequality (a>b) **decreases**; of less inequality **increases** — both drift towards 1 : 1.
- a : b = c : d ⇒ **ad = bc** (cross-product) — the backbone of every ratio equation.

Ratio of three or more terms

a : b : c means the three parts are **ak, bk, ck**. Simplify by dividing every term by their common HCF.

18 : 24 : 30 → ÷6 → **3 : 4 : 5** · 10 : 15 : 25 → ÷5 → **2 : 3 : 5**

Mini-example — using the k-trick

Two numbers are in ratio 5 : 8 and their sum is 91. Let them be 5k, 8k.

$5k + 8k = 13k = 91 \rightarrow k = 7$ ∴ numbers = **35 and 56**. ✅

Try these

- Simplify 121 : 132 → ÷11 → **11 : 12**
- Greater of 7 : 9 or 8 : 11 → 7×11=77 vs 9×8=72 → **7 : 9**
- 45 : 60 : 75 → ÷15 → **3 : 4 : 5**
- Ratio 2 : 7, sum 63 → 9k = 63, k = 7 → **14 and 49**

✓ Whenever a problem gives a ratio, immediately rename the parts with a common multiplier **k** — then every condition becomes a simple linear equation.

② Types of Ratios

Derived ratios of a : b

- **Duplicate** ratio = $a^2 : b^2$
- **Sub-duplicate** ratio = $\sqrt{a} : \sqrt{b}$
- **Triplicate** ratio = $a^3 : b^3$
- **Sub-triplicate** ratio = $\sqrt[3]{a} : \sqrt[3]{b}$
- **Inverse / Reciprocal** ratio = $b : a$ (i.e. $1/a : 1/b$)
- **Compound** ratio of $a : b$ & $c : d$ = $ac : bd$ (multiply term-wise)

Type	Rule	Example
Duplicate	$a^2 : b^2$	$3 : 4 \rightarrow 9 : 16$
Sub-duplicate	$\sqrt{a} : \sqrt{b}$	$9 : 16 \rightarrow 3 : 4$
Triplicate	$a^3 : b^3$	$2 : 3 \rightarrow 8 : 27$
Sub-triplicate	$\sqrt[3]{a} : \sqrt[3]{b}$	$8 : 27 \rightarrow 2 : 3$
Inverse	$b : a$	$5 : 7 \rightarrow 7 : 5$
Compound	$ac : bd$	$2:3$ & $4:5 \rightarrow 8:15$

Short examples — read them aloud

- Duplicate of $5 : 6 = 25 : 36$ · Sub-duplicate of $25 : 49 = 5 : 7$
- Triplicate of $1 : 2 = 1 : 8$ · Sub-triplicate of $27 : 64 = 3 : 4$
- Inverse of $9 : 4 = 4 : 9$ · Compound of $3 : 5$ & $10 : 9 = 30 : 45 =$ **2 : 3**

Do **not** confuse duplicate ratio ($a^2 : b^2$) with double ratio ($2a : 2b = a : b$). Squaring the terms is **not** the same as doubling them!

Compound Ratio — worked

Worked example — compound of 2 : 3, 6 : 7 and 14 : 5

Step 1 — multiply antecedents: $2 \times 6 \times 14 = 168$.

Step 2 — multiply consequents: $3 \times 7 \times 5 = 105$.

Step 3 — $168 : 105 \rightarrow \div 21 \rightarrow$ **8 : 5**. ✅

③ Combining Ratios (chaining)

Making a common link term

- Given a : b and b : c → make the **common b equal** (take its LCM), then read off **a : b : c**.
- For A : B = a : b, B : C = c : d, C : D = e : f → **A : B : C : D = ace : bce : bde : bdf**.

Worked example — a : b = 2 : 3 and b : c = 4 : 5

Step 1 — b appears as 3 and 4; LCM(3, 4) = 12.

Step 2 — scale a : b by 4 → 8 : 12; scale b : c by 3 → 12 : 15.

Answer → a : b : c = **8 : 12 : 15**. ✅

Three-ratio chain — A:B=2:3, B:C=4:5, C:D=6:7

A : B : C : D = (2·4·6) : (3·4·6) : (3·5·6) : (3·5·7) = 48 : 72 : 90 : 105.

÷3 → **16 : 24 : 30 : 35** (check 16:24=2:3, 24:30=4:5, 30:35=6:7 ✓)

Try these

- a:b=3:4, b:c=8:9 → b: LCM(4,8)=8 → 6:8, 8:9 → **6 : 8 : 9**
- a:b=5:6, b:c=9:10 → b: LCM(6,9)=18 → 15:18, 18:20 → **15 : 18 : 20**
- a:b=1:2, b:c=3:4 → b: LCM(2,3)=6 → 3:6, 6:8 → **3 : 6 : 8**

*Only the **shared term** must be made equal — scale each ratio by the factor that lifts its b to the LCM. The outer terms follow along.*

④ Dividing in a Given Ratio ⊕

Share formula

Each share = **(its ratio term / sum of all terms) × Total.**

Split Total into (a + b + c) equal parts, then hand out a, b, c of them.

Worked example — divide ₹720 in 2 : 3 : 4

Step 1 — sum of terms = 2 + 3 + 4 = **9** parts → one part = 720 / 9 = ₹80.

Step 2 — shares = 2×80, 3×80, 4×80.

Answer → **₹160, ₹240, ₹320** (160+240+320 = 720 ✓)

Worked — a difference is given, not the total

Divide a sum in 5 : 8 so that the **larger share exceeds the smaller by ₹120.**

Difference of parts = 8 − 5 = 3 → 3 parts = ₹120 → 1 part = ₹40.

Shares = 5×40, 8×40 = **₹200 and ₹320**; total sum = ₹520. ✅

Try these

- ₹1000 in 1 : 4 → 5 parts → **₹200 & ₹800**
- ₹1500 in 2 : 3 : 5 → 10 parts, 1 part=150 → **₹300, ₹450, ₹750**
- 96 sweets in 3 : 5 → 8 parts, 1 part=12 → **36 & 60**
- ₹840 in 3 : 4 : 5 → 12 parts, 1 part=70 → **₹210, ₹280, ₹350**

✓ If only a **part**, a **difference** or an **excess** is given, first find the value of **one part** — then every share follows instantly.

⑤ Proportion ∺

Definition

Four quantities are in proportion when **a : b :: c : d**, meaning **a/b = c/d**.

- a, d = **extremes** (outer) · b, c = **means** (inner).
- **Product of extremes = product of means → a × d = b × c.**

Example – check if 3, 4, 9, 12 are in proportion

Extremes 3 × 12 = 36; means 4 × 9 = 36. Equal ∴ **3 : 4 :: 9 : 12** is a true proportion. ✅

Worked example – find the missing extreme

If 8 : 12 :: 10 : x, find x.

Rule → 8 × x = 12 × 10 → 8x = 120.

Answer → x = **15**. ✅

Two flavours of proportion

Continued proportion

a : b :: b : c — the middle term repeats. Here **b^2 = ac**.

4, 6, 9 → 6^2=36=4×9 ✓

Discrete proportion

a : b :: c : d — all four terms different.

2 : 5 :: 6 : 15 ✓

💡 To find any one unknown in a proportion, always fall back on **extremes** × = **means** ×. It converts the proportion into a one-line equation.

⑥ Mean, Third & Fourth Proportional

Formulas to memorise

- **Mean proportional** between a and b = $\sqrt{ab}$ (from $a : x :: x : b$)
- **Third proportional** to a, b = b^2/a (from $a : b :: b : x$)
- **Fourth proportional** to a, b, c = bc/a (from $a : b :: c : x$)
- **Continued proportion** $a : b : c \Rightarrow b^2 = ac$ (b is the mean prop. of a & c)

Short examples for each

- Mean proportional of 4 & 9 = $\sqrt{36}$ = **6**
- Third proportional to 4, 6 = $6^2/4 = 36/4$ = **9**
- Fourth proportional to 3, 4, 6 = $(4\times6)/3$ = **8**
- In continued prop. 5, x, 20 → $x = \sqrt{(5\times20)} = \sqrt{100}$ = **10**

Worked example — third proportional to 9 and 12

Set-up — $9 : 12 :: 12 : x \rightarrow 9x = 12 \times 12 = 144$.

Answer → $x = 144/9 =$ **16**. ✅

Worked example — fourth proportional to 5, 8, 15

Set-up — $5 : 8 :: 15 : x \rightarrow 5x = 8 \times 15 = 120$.

Answer → $x =$ **24**. ✅

Mean proportional uses a **square-root** (term repeats twice); third & fourth proportional come from the **last unknown extreme**. Match the pattern before plugging in.

⑦ Properties of a Proportion

If **$a/b = c/d$**, the following transformed ratios are also equal — classic SSC shortcuts.

The five identities

- **Invertendo** → $b/a = d/c$
- **Alternendo** → $a/c = b/d$
- **Componendo** → $(a+b)/b = (c+d)/d$
- **Dividendo** → $(a-b)/b = (c-d)/d$
- **Componendo–Dividendo** → $(a+b)/(a-b) = (c+d)/(c-d)$

Quick sanity check with $a/b = 3/2$

Invertendo → $2/3$ · Alternendo → $a/c = b/d$ (both $= 3/3$ if c:d also 3:2).
Componendo → $(3+2)/2 = 5/2$ · Dividendo → $(3-2)/2 = 1/2$ · C–D → $5/1 =$ **5**.

Worked — componendo-dividendo to find x : y

Given $(x + y)/(x - y) = 7/5$, find $x : y$.
Apply C–D → add & subtract numerator/denominator:
$[(x+y)+(x-y)] \,/\, [(x+y)-(x-y)] = (7+5)/(7-5)$.
→ $2x/2y = 12/2 = 6$ ∴ **$x : y = 6 : 1$**. ✅ (check: $7/5$ ✓)

✓ When a fraction of the form $(p+q)/(p-q)$ is given, **componendo-dividendo** collapses it to p/q in one line — a huge time-saver.

⑧ Equal-Ratios Property

Adding across equal ratios

If $a/b = c/d = e/f = k$, then each ratio also equals

$$(a + c + e)/(b + d + f) = (pa + qc + re)/(pb + qd + rf)$$

for any weights p, q, r — the value stays k.

Simple example

$2/3 = 4/6 = 6/9$ (all $= 2/3$). Add tops and bottoms:

$(2+4+6)/(3+6+9) = 12/18 =$ **$2/3$**. Same value ✓

Worked example

If $a/3 = b/5 = c/7$ and $a + b + c = 90$, find a, b, c.

Step 1 — each ratio $= (a+b+c)/(3+5+7) = 90/15 =$ **6** $(= k)$.

Step 2 — $a = 3k = 18$, $b = 5k = 30$, $c = 7k = 42$.

Answer → **$a = 18$, $b = 30$, $c = 42$** (sum $= 90$ ✓)

Weighted version — try it

Given $a/2 = b/3 = c/4 = k$. Then $(2a + 3b + c)\ /\ (2\cdot2 + 3\cdot3 + 4)$ with weights matching…

Simplest use → $(a+b+c)/9 = k$, so if $a+b+c = 27$, $k = 3$ → **$a=6$, $b=9$, $c=12$**.

When several fractions are equal, set them all to a single letter **k**. Then every numerator is (its denominator) × k — the whole system solves in one step.

⑨ Direct & Inverse Variation $\propto$

Direct: $y \propto x$

$y = kx \rightarrow y/x = k$ stays constant.

More → more. Both rise together.

Inverse: $y \propto 1/x$

$xy = k$ stays constant.

More → less. One rises, other falls.

Worked example — direct variation

y is directly proportional to x; $y = 12$ when $x = 3$. Find y when $x = 5$.

Step 1 — $k = y/x = 12/3 = 4$.

Step 2 — $y = kx = 4 \times 5 =$ **20**. ✅

Worked example — inverse variation

6 workers finish a job in 10 days. How long for 4 workers (same rate)?

Step 1 — workers × days = $k = 6 \times 10 = 60$.

Step 2 — $4 \times$ days $= 60 \rightarrow$ days = **15**. ✅

Spot the type

- Cost ∝ quantity bought → **direct**.
- Speed ∝ 1/time (fixed distance) → **inverse**.
- Men ∝ 1/days (fixed work) → **inverse**.
- Distance ∝ time (fixed speed) → **direct**.

Direct → keep the ratio **y/x** fixed. Inverse → keep the **product xy** fixed. Decide the type first, then equate constants.

⑩ Applications — Coins & Mixtures

Coins — number vs value

If coins of different denominations are in a **number ratio**, their **value ratio** = (number × denomination).

₹1, 50p, 25p equal in number → value ratio = 1 : 0.5 : 0.25 = 4 : 2 : 1.

Worked — coins problem

A bag has ₹1, 50p and 25p coins in the number ratio **2 : 3 : 4**; total value ₹90. Find the count of each.

Step 1 — let counts be 2k, 3k, 4k.

Step 2 — value = 2k(1) + 3k(0.5) + 4k(0.25) = 2k + 1.5k + 1k = 4.5k.

Step 3 — 4.5k = 90 → k = 20.

Answer → **40 one-rupee, 60 fifty-paise, 80 twenty-five-paise** coins. ✅

Worked — mixture "ratio becomes"

Milk : water = 5 : 3. On adding 4 L water it becomes 5 : 4. Find the milk.

Step 1 — milk = 5k, water = 3k. Milk is unchanged.

Step 2 — 5k / (3k + 4) = 5/4 → 20k = 15k + 20 → 5k = 20 → k = 4.

Answer → milk = 5k = **20 L** (water 12 L → 16 L). ✅

✓ In "something is added" mixture sums, the **untouched** quantity keeps its k. Write the new ratio as an equation and solve for k.

Ages & "Ratio Changes" Problems

Worked — ages

Present ages of A and B are in ratio **5 : 7**. After 6 years the ratio becomes **3 : 4**. Find their present ages.

Step 1 — let ages be $5k$ and $7k$.

Step 2 — $(5k + 6) / (7k + 6) = 3/4 \rightarrow 4(5k+6) = 3(7k+6)$.

Step 3 — $20k + 24 = 21k + 18 \rightarrow k = 6$.

Answer → A = 5k = **30 yr**, B = 7k = **42 yr** (36 : 48 = 3 : 4 ✓)

Worked — both terms increased

Two numbers are in ratio 3 : 4. If each is increased by 6, the ratio becomes 4 : 5. Find them.

Step 1 — numbers $3k$, $4k \rightarrow (3k+6)/(4k+6) = 4/5$.

Step 2 — $5(3k+6) = 4(4k+6) \rightarrow 15k + 30 = 16k + 24 \rightarrow k = 6$.

Answer → **18 and 24** (24 : 30 = 4 : 5 ✓)

Short — number decreased

Ratio 7 : 5; subtract 4 from each → $(7k-4)/(5k-4) = 5/3 \rightarrow 3(7k-4)=5(5k-4) \rightarrow 21k-12=25k-20 \rightarrow 4k=8 \rightarrow k=2 \rightarrow$ **14 & 10**.

"Add / subtract the same amount to both" ⟶ write parts as 3k, 4k, apply the new ratio and cross-multiply. One equation, one unknown k.

Income, Expenditure & Savings

The key relation

Savings = Income − Expenditure. Use a common k for each ratio and link them through savings.

Worked — two people, equal savings

Incomes of A, B are 3 : 4; expenditures 5 : 7. Each **saves ₹4000**. Find incomes.

Step 1 — incomes = 3x, 4x; savings = 4000 each.

Step 2 — expenditures = 3x − 4000 and 4x − 4000, in ratio 5 : 7.

Step 3 — $(3x-4000)/(4x-4000) = 5/7 \rightarrow 7(3x-4000) = 5(4x-4000)$.

Step 4 — $21x - 28000 = 20x - 20000 \rightarrow x = 8000$.

Answer → incomes = **₹24000 and ₹32000** (exp 20000 : 28000 = 5 : 7 ✓)

Short — income & savings rate

A's income : expenditure = 5 : 4. So savings = 5 − 4 = 1 part = 20% of income. If income = ₹15000, savings = 15000/5 = **₹3000**.

Try these

- Income:exp = 7 : 5, income ₹21000 → exp = 21000×5/7 = **₹15000**, saves ₹6000.
- Salary rises 4 : 5, spending 3 : 4; both save the same → solve as above with the k-trick.

✓ Give **incomes one letter** and **expenditures another** only if their ratios differ — then bridge them with the savings condition to eliminate a variable.

Applications — Fully Worked

Q — partnership share

P and Q invest in the ratio 4 : 5 for the same time; profit = ₹5400. Q's share?

Profit splits in the investment ratio 4 : 5 (9 parts). Q = 5/9 × 5400 = **₹3000** (P = ₹2400). ✅

Q — coins of two kinds

A purse has ₹5 and ₹2 coins in ratio 3 : 5; total value ₹100. Number of ₹2 coins?

Value = 5(3k) + 2(5k) = 15k + 10k = 25k = 100 → k = 4. ₹2 coins = 5k = **20** (₹5 coins = 12). ✅

Q — ages, sum given

Father : son ages = 7 : 3; sum of ages = 50. Ages after 5 years?

7k + 3k = 10k = 50 → k = 5 → now 35 & 15. After 5 yr → **40 and 20** (ratio 2 : 1). ✅

Q — income & savings

Incomes A : B = 5 : 6, expenditures 3 : 4; each saves ₹2000. A's income?

(5x−2000)/(6x−2000) = 3/4 → 4(5x−2000)=3(6x−2000) → 20x−8000=18x−6000 → 2x=2000 → x=1000. A = 5x = **₹5000** (B = ₹6000). ✅

Q — mixture top-up

Alcohol : water = 4 : 3 in 21 L. How much water to make it 4 : 5?

4k+3k=7k=21 → k=3 → alcohol 12 L, water 9 L. Need 12/(9+w)=4/5 → 60=36+4w → w = **6 L**. ✅

💡 *Every one of these reduces to: name the parts with k, translate the condition into one equation, solve, back-substitute.*

Solved Examples ∑

1. Divide ₹5000 among A, B, C in 2 : 3 : 5.
Parts = 10, one part = 500 → **₹1000, ₹1500, ₹2500**.

2. Mean proportional between 8 and 32.
$\sqrt{(8 \times 32)} = \sqrt{256} =$ **16**.

3. a : b = 3 : 4 and b : c = 8 : 9. Find a : b : c.
b: LCM(4, 8) = 8 → a : b = 6 : 8, b : c = 8 : 9 → **6 : 8 : 9**.

4. Fourth proportional to 5, 8, 15.
5 : 8 :: 15 : x → x = (8 × 15) / 5 = **24**.

5. Two numbers are 5 : 8; adding 9 to each makes them 8 : 11. Find them.
(5k+9) / (8k+9) = 8 / 11 → 11(5k+9) = 8(8k+9) → 55k+99 = 64k+72 → 9k = 27 → k = 3 → **15 and 24**.

6. If (a+b) : (a−b) = 5 : 1, find a : b.
Componendo–dividendo backwards → a / b = (5+1) / (5−1) = 6 / 4 = **3 : 2**.

✓ *Notice how examples 5 and 6 use the very same k-trick and componendo–dividendo you met earlier — the toolkit is small; practise until it is reflex.*

Practice Set

1. Simplify 84 : 105.
2. Duplicate ratio of 5 : 6 = ?
3. Sub-duplicate of 16 : 25 = ?
4. Triplicate ratio of 2 : 5 = ?
5. Compound of 2 : 3 & 6 : 7 = ?
6. Divide ₹880 in 3 : 5.
7. Mean proportional of 9 & 16 = ?
8. Third proportional to 5 & 10 = ?
9. Fourth proportional to 4, 6, 10 = ?
10. a:b=2:3, b:c=6:7 → a:b:c = ?
11. x : y = 5 : 3; (x+y):(x−y) = ?
12. Ages 4 : 5; after 5 yr 5 : 6. Now?
13. Milk:water 7:2; +6 L water → 7:5. Milk?
14. Incomes 3:4, exp 5:7, save ₹4000. A?
15. ₹1,50p,25p in 2:3:4, total ₹90. 25p count?

Answer Key — with working

1→ 4 : 5 (÷21) · **2→ 25 : 36** (5^2:6^2) · **3→ 4 : 5** (√16:√25) · **4→ 8 : 125** (2^3:5^3) · **5→ 4 : 7** (12:21) · **6→ ₹330 & ₹550** (8 parts, 1=110) · **7→ 12** (√144) · **8→ 20** (10^2/5) · **9→ 15** (6×10/4) · **10→ 4 : 6 : 7** (b=6) · **11→ 4 : 1** (8:2) · **12→ 20 & 25** (k=5) · **13→ 14 L** (7k/(2k+6)=7/5, k=2) · **14→ ₹24000** (x=8000) · **15→ 80** (4.5k=90, k=20).

Quick Revision

- ✓ a : b = a/b; rename parts as ak, bk. Duplicate a^2:b^2, sub-dup √a:√b, triplicate a^3:b^3, compound ac:bd.
- ✓ Proportion a:b::c:d → ad = bc. Mean √(ab); third b^2/a; fourth bc/a; continued b^2=ac.
- ✓ Componendo–dividendo (a+b)/(a−b)=(c+d)/(c−d); equal ratios → each = (Σnum)/(Σden).

☆ Ratios & proportions — compare with confidence! ☆