Cubes & Dice

~ Reasoning · SSC Exams ~

← circled = key tricks!

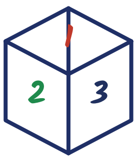
Cubes & Dice tests your **3-D visualisation** — how a cube's faces relate when it is rotated, viewed from different angles, unfolded flat, or physically painted & sliced into smaller cubes. Learn the few core rules below and every question becomes mechanical.

1 Faces, Edges, Vertices & the Sum-7 Rule

Every cube / die has **6 faces, 12 edges, 8 vertices** (corners). A standard die's faces carry the numbers 1–6.

Standard-Dice Rule — the numbers on **opposite faces always add up to 7**:

1 ↔ **6** **2** ↔ **5** **3** ↔ **4**



The 3 faces you ***see*** *(top, front, right) and the 3* ***hidden*** *faces (bottom, back, left) are always exact opposite pairs. On a standard die, if you can read the top you instantly know the bottom (7 − top).*

Worked example

A standard dice is thrown and shows **4** on top. Find the bottom face, and the sum of the **4 side (vertical) faces**.

Step 1 — bottom = 7 − top = 7 − 4 = **3**.

Step 2 — total of all 6 faces = 1+2+3+4+5+6 = 21.

Answer → side faces = 21 − top − bottom = 21−4−3 = **14**. ✅

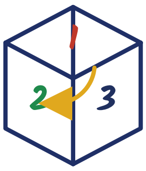
Tip → the Sum-7 rule works **only for a standard dice**. Never assume it for an "ordinary" dice — the question must say so, or you must derive opposite faces from the given views (pages ahead)!

② Standard vs Ordinary Dice

Rule

Standard dice — opposite faces always sum to 7; conventionally the numbers 1, 2, 3 are arranged **anti-clockwise** around their common corner (viewed from outside that corner).

Ordinary (non-standard) dice — the Sum-7 rule does **NOT** apply. Opposite faces must be worked out purely from the position(s) the question gives you.



The curved arrow shows the **anti-clockwise** flow 1→2→3 around their shared corner — this is the standard-dice convention used when a question says "a standard dice is shown" without giving you a picture.

Worked example

In a standard dice, 1, 2, 3 lie anti-clockwise about a common corner. If **1 is on top** and **2 is in front**, which number is on the **right** face?

Step 1 — going anti-clockwise around that corner the order is 1 → 2 → 3.

Step 2 — top=1, front=2 are the first two steps, so the next face in the anti-clockwise chain (right) must be 3.

Answer → right face = **3**. ✅

Try this — an ordinary dice

Question says: "the following dice are **ordinary**", and shows 4 on top, 5 in front, 2 on the right. Here you **cannot** assume 4↔3, 5↔2, 2↔5 (the standard pairs) — the opposite of 4 is simply whichever face never shows up alongside it in any given view. You must use the multi-position rules on the next two pages. ✅

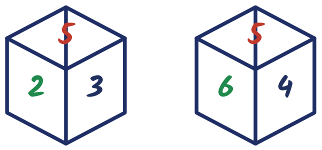
Some books use a **clockwise** 1-2-3 convention instead — if the question's picture disagrees with your assumed direction, trust the picture, not the memorised rule!

Safest habit → whenever two or more positions of the SAME dice are drawn out for you, treat it as **ordinary** and deduce opposite faces from the pictures — that method always works, standard or not.

③ Opposite Face – Common-Face Rule

Rule (One Common Face)

If the **same face is common** (same number, same position — e.g. Top in both) in two throws of a dice, and the other 4 visible numbers (2 per throw) are **all different**, then the ONE number that appears in **neither** throw is the face **opposite** the common face.



Position 1 Position 2

Worked example

Position 1: Top=5, Front=2, Right=3. Position 2: Top=5, Front=6, Right=4. Find the face opposite 5.

Step 1 — Top=5 is common to both throws.

Step 2 — other visible numbers: 2, 3 (Pos 1) and 6, 4 (Pos 2) — all four are different, so the rule applies. Numbers seen so far: 5,2,3,6,4 — only **1** is missing.

Answer → 1 never appears with 5, so 1 is opposite 5. **Opposite of 5 = 1.** ✅

Try this

Pos 1: Top=1, Front=2, Right=6. Pos 2: Top=1, Front=3, Right=4. Numbers seen: 1,2,6,3,4 — missing number is **5**. ∴ opposite of 1 = **5**. ✅

This rule needs the same face TRULY common (same position) in both throws, and all 4 remaining numbers distinct. If a number repeats, use the Two-Common-Faces rule (next page) instead.

④ Two-Common-Faces Rule ✕+

Rule

If **two numbers are common** between two throws (they simply swap positions, e.g. Top↔Front), the **two remaining, different** numbers — one from each throw — are **opposite** each other.

Worked example

Pos 1: Top=2, Front=3, Right=6. Pos 2: Top=3, Front=5, Right=2.

Step 1 — common numbers 2 & 3 appear in both throws (just swapped between Top/Front/Right).

Step 2 — remaining, different numbers: 6 (Pos 1) and 5 (Pos 2).

Answer → 6 is opposite 5. **Opposite of 6 = 5.** ✅

⑤ Elimination Method (3 Positions) ○○○

When 3 different positions are given, find the number opposite a face X by removing (from 1–6) every number that appears **alongside** X in any throw. The lone survivor is opposite X.

Worked example — find opposite of 1

Pos 1 shows 1,2,3 · Pos 2 shows 1,4,5 · Pos 3 shows 4,3,6.

Step 1 — in Pos 1, 1 is seen with 2 & 3 → so opposite(1) ≠ 2, 3.

Step 2 — in Pos 2, 1 is seen with 4 & 5 → so opposite(1) ≠ 4, 5. Remaining candidate from {1..6}: only **6** is left.

Answer → **opposite of 1 = 6.** (Pos 3, where 1 is hidden, indeed shows 3,4,6 — consistent!) ✅

Trap ⟶ never mix up "seen together = adjacent, not opposite" with "opposite = never seen together."
Cross out every number you see WITH your target face — the last one standing is its opposite.

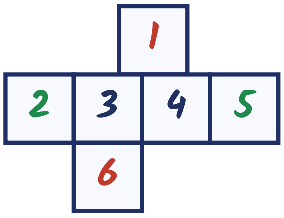
⑥ Nets / Unfolded Dice

3 Folding Rules

Rule 1 — two squares that **share an edge** (are adjacent) in the net can **never** be opposite faces after folding.

Rule 2 — in a **straight row of 4** squares, the **1st & 3rd** are opposite, and the **2nd & 4th** are opposite (like a belt: front-right-back-left).

Rule 3 — two flaps attached on **opposite sides** (top & bottom) of the **same square** become opposite faces after folding.



Row of 4 = **2,3,4,5** → (1st,3rd)=(2,4) opposite & (2nd,4th)=(3,5) opposite. Flaps **1** (top of 3) & **6** (bottom of 3) → opposite.

Worked example

Fold the net above into a cube. List all 3 opposite pairs.

Step 1 — row of 4 is 2-3-4-5: pair (1st,3rd) = 2↔4, pair (2nd,4th) = 3↔5.

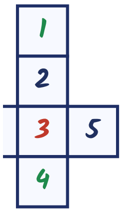
Step 2 — flaps 1 (top of square 3) & 6 (bottom of square 3) fold to top & bottom of the cube → 1↔6.

Answer → **1↔6, 2↔4, 3↔5**. ∴ this happens to also be a standard dice! ✅

Mentally fold ONE square (call it "front") and trace its 4 neighbours to right/left/top/bottom — whatever wraps around to the far side is the "back", opposite front.

⑦ More Dice Practice

Try this — another net



Vertical row of 4 = **1,2,3,4** → (1st,3rd)=1↔3 and (2nd,4th)=2↔4. Flaps **5** & **6** attach left/right of square 3 → 5↔6.
Answer: 1↔3, 2↔4, 5↔6.

Quick fact — total pips on a standard dice

Sum of numbers on all 6 faces = 1+2+3+4+5+6 = **21**. Useful whenever a question asks for the total number of dots visible or hidden.

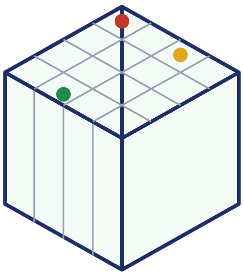
Worked example — two dice glued together

Two identical standard dice are glued face-to-face so that a **6** touches a **6**. Find the total of all numbers now visible on the outer surface of the joined solid.
Step 1 — each single dice totals 21 dots on 6 faces; two dice = 21+21 = 42.
Step 2 — the two glued (touching) faces are both hidden: 6 + 6 = 12 dots disappear from view.
Answer → visible total = 42 − 12 = **30**.

For "n dice joined in a row" questions: total visible = 21n − (sum of all touching-face pairs), since each internal joint hides exactly 2 faces.

⑧ Painted Cube — Setup & Formulas

*A big cube of side **n** units is painted on **all 6 outer faces**, then sawn into n^3 unit cubes. Every unit cube is classified by **how many painted faces** it shows.*



- **Corner** — 3 faces painted
- **Edge** — 2 faces painted
- **Face** — 1 face painted
- **Inside** — 0 faces painted

Master Formulas (side = n, $n \geq 2$)

Corner cubes (3 faces) → **8** — always, for any $n \geq 2$.

Edge cubes (2 faces) → $\mathbf{12(n-2)}$

Face cubes (1 face) → $\mathbf{6(n-2)^2}$

Inside cubes (0 faces) → $\mathbf{(n-2)^3}$

Worked example — $n = 4$

Step 1 — corner = 8. Edge = $12(4-2) = 12\times2 = 24$.

Step 2 — face = $6(4-2)^2 = 6\times4 = 24$. Inside = $(4-2)^3 = 2^3 = 8$.

Answer → total = $8+24+24+8 = \mathbf{64} = 4^3$ ✓ ✅

💡 *All four counts always add back to n^3 — use this as an instant self-check after every calculation!*

⑨ Worked Examples — Different n

n = 5 (real SSC Q. style)

A cube of side 5 cm is painted red on all faces & cut into 1 cm cubes. How many have exactly **2** painted faces, and how many have **none**?

Step 1 — edge (2 faces) = $12(5-2) = 12\times 3 =$ **36**.

Answer → inside (0 faces) = $(5-2)^3 = 3^3 =$ **27**. ✅

n = 6

Cube of side 6 units, painted & cut into unit cubes. Find the cubes with exactly **1** painted face.

Step 1 — face (1 face) = $6(6-2)^2 = 6\times 16 =$ **96**.

Answer → **96** unit cubes have exactly one painted face. ✅

n = 3 (smallest interesting case)

Cube of side 3, painted & cut — edge = $12(1) = 12$, face = $6(1)^2 = 6$, inside = $1^3 = 1$, corner = 8. Total = $8+12+6+1 =$ **27** $= 3^3$ ✓. ∴ every unit cube except the exact centre has at least one painted face. ✅

Try these — quick answers

$n=2 \rightarrow$ corner=**8**, rest = 0	$n=7 \rightarrow$ edge=12(5)=**60**
$n=8 \rightarrow$ inside=6^3=**216**	$n=10 \rightarrow$ face=$6(8)^2$=**384**

💡 At n=2 the cube is made of ONLY corner cubes (8 of them, each with 3 painted faces) — a favourite "trick" SSC option to test if you blindly apply edge/face formulas without checking $n-2 = 0$.

⑩ Special Variants & Cuboids

Variant A — only 2 opposite faces painted (top & bottom)

No cube gets 2 or 3 painted faces (painted faces never touch). 1-face cubes = $\mathbf{2n^2}$ (top & bottom layers). 0-face cubes = $\mathbf{n^2(n-2)}$.

$n=4$, only top & bottom painted → unpainted = $n^2(n-2) = 16\times2 =$ **32**. Check: painted = $2n^2=32$, $32+32=64=4^3$ ✓ ✅

Variant B — only 4 side faces painted (top & bottom left bare)

2-face cubes lie on the 4 vertical edges = $\mathbf{4n}$. 1-face cubes = $\mathbf{4n(n-2)}$. 0-face cubes = $\mathbf{n(n-2)^2}$.

$n=3$ → 2-face = $4\times3=12$, 1-face = $4\times3\times1=12$, 0-face = $3\times1^2=3$. Check: $12+12+3=27=3^3$ ✓ ✅

Variant C — different colour on every face: only the **8 corner cubes** can show 3 different colours (one per meeting face) — every other cube shows at most the colours of the faces it actually touches.

Cuboid Cuts (length × breadth × height)

Corner = **8** · Edge = $\mathbf{4(l-2)+4(b-2)+4(h-2)}$

Face = $\mathbf{2[(l-2)(b-2)+(b-2)(h-2)+(h-2)(l-2)]}$ · Inside = $\mathbf{(l-2)(b-2)(h-2)}$

Example — cuboid $5\times4\times3$ units, painted & cut: corner=8, edge=$4(3)+4(2)+4(1)=24$, face=$2[6+2+3]=22$, inside=$3\times2\times1=6$. Total=$8+24+22+6=$**60**$=5\times4\times3$ ✓ ✅

Master Table & Quick Revision

Painted-Cube Table (n = 2 to 8)

n	Total n^3	Corner (3f)	Edge (2f)	Face (1f)	Inside (0f)
2	8	8	0	0	0
3	27	8	12	6	1
4	64	8	24	24	8
5	125	8	36	54	27
6	216	8	48	96	64
7	343	8	60	150	125
8	512	8	72	216	216

Quick Pattern Gallery — one-glance revision

Sum-7: 1↔6, 2↔5, 3↔4

1 common face: missing number = opposite

2 common faces: remaining 2 → opposite

Net: adjacent squares never opposite

Net row of 4: 1st-3rd & 2nd-4th opposite

Corner cubes: always **8**, any $n \geq 2$

Edge cubes: $12(n-2)$

Face cubes: $6(n-2)^2$

Inside cubes: $(n-2)^3$

Self-check: all 4 counts sum to n^3

Memorise the row for n=4,5,6 cold — SSC papers reuse these three sizes far more often than any other.

Exercise — 12 Questions

Mixed Dice + Painted Cube questions — solve, then check the Answer Key on the next page.

Q1. Standard dice shows 5 on top. What is on the bottom?
(1) 1 (2) 2 (3) 3 (4) 4

Q2. Pos1: Top=4,Front=1,Right=2. Pos2: Top=4,Front=6,Right=3. Which number is opposite 4?
(1) 1 (2) 5 (3) 6 (4) 2

Q3. Pos1: Top=2,Front=3,Right=6. Pos2: Top=3,Front=5,Right=2. Number opposite 6?
(1) 2 (2) 3 (3) 5 (4) 1

Q4. Standard dice: 1,2,3 anti-clockwise about a corner. Top=1, Front=2. Right face?
(1) 3 (2) 4 (3) 5 (4) 6

Q5. Net row of 4 = 2,3,4,5 (in order); flaps 1 (top of 3), 6 (bottom of 3). Which opposite set is correct?
(1) 1-6,2-3,4-5 (2) 1-6,2-4,3-5 (3) 1-2,3-4,5-6 (4) 2-6,3-4,1-5

Q6. Sum of numbers on all 6 faces of a standard dice?
(1) 15 (2) 18 (3) 20 (4) 21

Q7. Cube of side 5, painted & cut into unit cubes. Cubes with exactly 2 painted faces?
(1) 24 (2) 36 (3) 48 (4) 12

Q8. Same cube (n=5). Cubes with NO painted face?
(1) 9 (2) 27 (3) 18 (4) 36

Q9. Cube of side 6, painted & cut. Cubes with exactly 1 painted face?
(1) 64 (2) 96 (3) 48 (4) 86

Q10. Cuboid 4×3×2 units, painted & cut. Cubes with exactly 2 painted faces?
(1) 16 (2) 8 (3) 20 (4) 12

Q11. Cube of side 7, painted & cut. Cubes with exactly 3 painted faces?
(1) 6 (2) 7 (3) 8 (4) 12

Q12. Cube of side 4, only top & bottom painted, then cut into unit cubes. Cubes with NO paint?
(1) 16 (2) 32 (3) 8 (4) 24

✓ Solve every question on paper first — sketch the dice/cube, mark faces — then flip the page to check.

✅ Answer Key

Q1 → 2 — Sum-7 rule: $7-5=2$.

Q2 → 5 — common-face rule: numbers seen with 4 are 1,2,6,3; missing = 5.

Q3 → 5 — two-common-faces rule: common 2&3, remaining 6↔5.

Q4 → 3 — anti-clockwise chain 1(top)→2(front)→3(right).

Q5 → 1-6,2-4,3-5 — row-of-4 rule (1st-3rd, 2nd-4th) + flap rule.

Q6 → 21 — $1+2+3+4+5+6=21$.

Q7 → 36 — edge = $12(5-2) = 12\times3 = 36$.

Q8 → 27 — inside = $(5-2)^3 = 27$.

Q9 → 96 — face = $6(6-2)^2 = 6\times16 = 96$.

Q10 → 12 — edge = $4(4-2)+4(3-2)+4(2-2) = 8+4+0 = 12$.

Q11 → 8 — corner cubes are always 8, for any $n\geq2$.

Q12 → 32 — unpainted = $n^2(n-2) = 16\times2 = 32$.

Score yourself

10–12 correct → **excellent**, Cubes & Dice is your strength! 6–9 → **good**, revisit pages 3–9. Below 6 → **redo** the folding & formula boxes slowly.

☆ *Turn every face, crack every cube!* ☆